

Hong Kong Mathematics Olympiad (2017/18)

Heat Event (Individual)

香港数学竞赛 (2017/18)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

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Part A

1. 若 a 及 b 均为实数，求 $a^2 + b^2 + 12a - 8b + 2018$ 的最小值。

If a and b are real numbers, find the minimum value of $a^2 + b^2 + 12a - 8b + 2018$.

2. 设 a 及 k 均为常数。若 $(6x^3 + ax^2 + 7x - 3) \div (2x^2 + kx - 1)$ 的商和余式分别为 $3x + 5$ 及 $-5x + 2$ ，求 a 的值。

Let a and k be constants. If the quotient and the remainder of $(6x^3 + ax^2 + 7x - 3) \div (2x^2 + kx - 1)$ are $3x + 5$ and $-5x + 2$ respectively, find the value of a .

3. 在编制某杂志中每页的页码时，总共用去了 2,046 个数字，问该杂志总共有多少页？（假设该杂志第一页的页码是 1。）

In numbering the pages of a magazine, 2,046 digits were used. How many pages are there in the magazine? (Assume the page number of the magazine starts from 1.)

4. 解 $\log\left(1 + \frac{1}{1}\right) + \log\left(1 + \frac{1}{2}\right) + \log\left(1 + \frac{1}{3}\right) + \dots + \log\left(1 + \frac{1}{n}\right) = 5$ 。

Solve $\log\left(1 + \frac{1}{1}\right) + \log\left(1 + \frac{1}{2}\right) + \log\left(1 + \frac{1}{3}\right) + \dots + \log\left(1 + \frac{1}{n}\right) = 5$.

5. 已知 $\frac{1 - 2^{-\frac{1}{x}}}{2^{-\frac{1}{x}} - 2^{-\frac{2}{x}}} = 4$ 。求 x 的值。

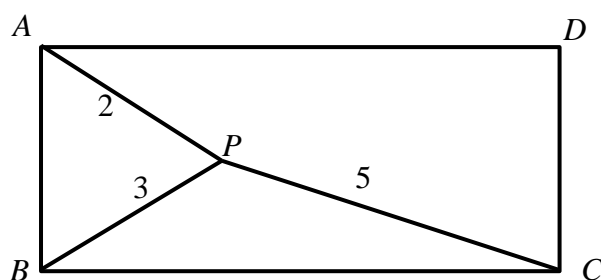
Given that $\frac{1 - 2^{-\frac{1}{x}}}{2^{-\frac{1}{x}} - 2^{-\frac{2}{x}}} = 4$. Find the value of x .

6. 若 x 为有理数, 求 x 的值满足联立方程 $\begin{cases} y = 2x^2 - 11x + 15 \\ y = 2x^3 - 17x^2 + 16x + 35 \end{cases}$ 。

If x is a rational number, find the value of x satisfying the simultaneous equations

$$\begin{cases} y = 2x^2 - 11x + 15 \\ y = 2x^3 - 17x^2 + 16x + 35 \end{cases}$$

7. 如图一所示, P 为长方形 $ABCD$ 内的一点, 使得 $PA = 2$, $PB = 3$ 及 $PC = 5$ 。求 PD 的长度。
As shown in Figure 1, P is a point inside a rectangle $ABCD$ such that $PA = 2$, $PB = 3$ and $PC = 5$. Find the length of PD .

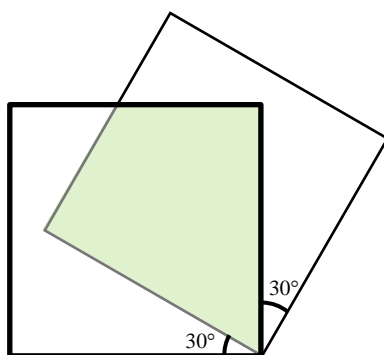


图一

Figure 1

8. 如图二所示, 两个边长为 x cm 的正方形于一角重迭。若两个正方形的非重迭部分与重迭部分面积的比是 $a : 1$, 求 a 的值。

As shown in Figure 2, two squares with side x cm coincide at one corner. If the ratio of the non-overlapping area to the overlapping area of the two squares is $a : 1$, find the value of a .

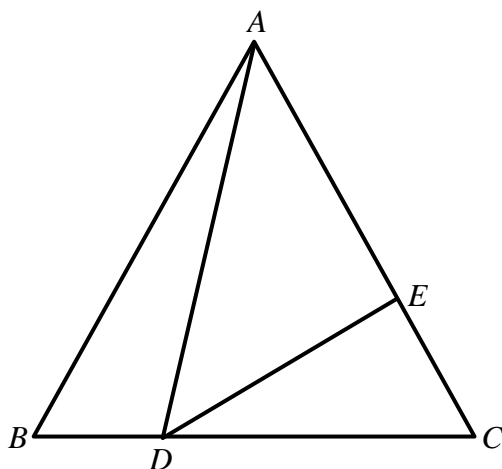


图二

Figure 2

9. 如图三所示, ABC 是一个等腰三角形, 其中 $AB = AC = 8$ 及 $BC = 4$ 。 D 及 E 分别为 BC 及 AC 上的点使得 $BD = 1$ 及 $\angle ABC = \angle ADE$ 。求 AE 的长度。

As shown in Figure 3, ABC is an isosceles triangle with $AB = AC = 8$ and $BC = 4$. D and E are points lying on BC and AC respectively such that $BD = 1$ and $\angle ABC = \angle ADE$. Find the length of AE .



图三

Figure 3

10. PQR 是一个三角形, 其中 $PQ = 13$ 、 $QR = 14$ 及 $PR = 15$ 。以 PR 为直径绘画出圆 C , C 相交 QR 于点 T 。求 $\triangle PTR$ 的面积。

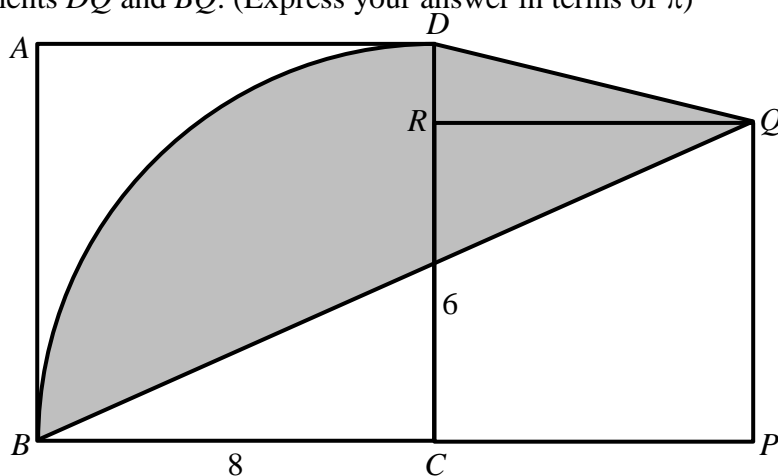
PQR is a triangle with $PQ = 13$, $QR = 14$ and $PR = 15$. The circle C is drawn with diameter PR . C intersects QR at a point T . Find the area of $\triangle PTR$.

11. 求 $3^x + 5 + \frac{36}{3^x + 4}$ 的最小值。

Find the minimum value of $3^x + 5 + \frac{36}{3^x + 4}$.

12. 如图四所示, $ABCD$ 及 $PQRC$ 为两个连接的正方形。以 C 为圆心及 BC 为半径绘画出弧 BD 。已知 $BC = 8$ 及 $RC = 6$ 。求弧 BD 及线段 DQ 与 BQ 所围成的区域的面积。(答案须以 π 表示)

As shown in Figure 4, two squares $ABCD$ and $PQRC$ are joined together. An arc BD is drawn with centre C and radius BC . Given that $BC = 8$ and $RC = 6$. Find the area of the region bounded by the arc BD , line segments DQ and BQ . (Express your answer in terms of π)



图四

Figure 4

13. 一个四位数可以透过把它的所有数字加起来, 变成另一个数。例如: 1234 可以变成 10, 因为 $1+2+3+4=10$ 。究竟从 1998 至 4998 (包括此两个数) 有多少个四位数经上述变换后不可以被 3 整除?

A 4-digit number can be transformed into another number by adding its digits. For example, 1234 can be transformed into 10 as $1+2+3+4=10$. How many numbers from 1998 to 4998 inclusive are **NOT** divisible by 3 after the above transformation?

14. 对任意实数 x ($x \neq 1$), 定义函数 $f(x) = \frac{x}{1-x}$ 及 $f \circ f(x) = f(f(x))$ 。求 $\underbrace{2017 f \circ f \circ f \circ \dots \circ f}_{2018 \text{ 个 } f}(2018)$ 的值。

For any real number x ($x \neq 1$), define a function $f(x) = \frac{x}{1-x}$ and $f \circ f(x) = f(f(x))$. Find the value of

$\underbrace{2017 f \circ f \circ f \circ \dots \circ f}_{2018 \text{ s } f}(2018)$.

15. 设 $N^2 = \overline{abcdefabc}$ 为一个 9 位整数, 其中 N 是 4 个相异质数的积及 a, b, c, d, e, f 均为非零数字且满足 $\overline{def} = 2 \times \overline{abc}$ 。求 N^2 的最小值。

Let $N^2 = \overline{abcdefabc}$ be a nine-digit positive integer, where N is the product of four distinct primes and a, b, c, d, e, f are non-zero digits that satisfy $\overline{def} = 2 \times \overline{abc}$. Find the least value of N^2 .

完

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